Numerical Comparison of Currency Price Prediction in Iran Using the Stochastic Black-Scholes Model with Different Noises

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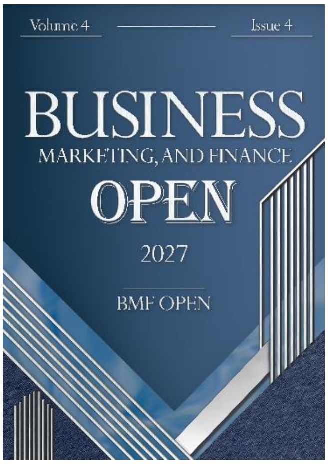

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Abstract: This study aims to compare the efficiency of mixed noise and Lévy noise as alternatives to Gaussian white noise in the stochastic Black–Scholes model for forecasting currency prices in Iran. The stochastic model under different noise processes was solved using the Euler–Maruyama numerical method. The model parameters were estimated from actual market data using the maximum likelihood estimation method, and a 95% confidence interval was calculated for the forecasted values. For this purpose, daily US dollar exchange-rate data covering the period from August 24 to December 13, 2024, were analyzed. The simulations were conducted using Maple software. To evaluate the accuracy and precision of the proposed approach, the US dollar exchange rate for December 22, 2024, was separately forecasted using the Euler–Maruyama method under Gaussian white noise, mixed noise, and Lévy noise. The corresponding simulation charts were also generated in Maple. The findings indicate that replacing Gaussian white noise and mixed noise with Lévy noise in the stochastic Black–Scholes model improves the efficiency of US dollar exchange-rate forecasting.

Keywords: Mixed Noise, Lévy Noise, Numerical Simulation, Ito Stochastic Differential Equation, Stochastic Black-Scholes Model, Confidence Interval, Efficiency.

1. Introduction

Forecasting exchange-rate movements is a central problem in financial mathematics, monetary economics, investment management, and risk analysis because currency prices respond simultaneously to macroeconomic conditions, market expectations, political events, regulatory interventions, liquidity constraints, and unanticipated external shocks. Unlike many physical systems whose trajectories may be approximated under relatively stable structural assumptions, foreign-exchange markets are characterized by nonlinear interactions, rapidly changing information flows, abrupt regime shifts, and substantial uncertainty. These characteristics become especially pronounced in emerging and inflationary economies, where exchange rates may be affected by monetary expansion, international sanctions, geopolitical tensions, changes in trade conditions, speculative behavior, and interventions in formal and informal currency markets. Consequently, deterministic forecasting models often fail to adequately represent the irregularity and uncertainty embedded in exchange-rate trajectories. A suitable predictive framework must therefore account not only for the expected trend of prices but also for the stochastic fluctuations and discontinuities that shape their observed behavior. In the present study, this issue is examined through the stochastic Black–Scholes model, in which Gaussian white noise, mixed noise, and Lévy noise are comparatively incorporated to predict currency prices in Iran. The original manuscript reports that the proposed models are numerically solved through the Euler–Maruyama method, their parameters are estimated from observed dollar prices, and their prediction errors are compared to evaluate the relative efficiency of alternative noise structures.

The increasing complexity of financial forecasting has led researchers to employ a wide range of statistical, computational, and machine-learning techniques. Recent studies have demonstrated that machine-learning models can identify nonlinear patterns in cryptocurrency and financial-price data that may not be adequately captured by conventional linear models. For example, comparative evaluations of machine-learning algorithms have shown that predictive performance varies substantially according to the data structure, selected variables, estimation horizon, and model architecture, indicating that no single forecasting method is uniformly superior across all markets and conditions [1]. Ensemble learning has also been proposed as a means of combining the strengths of several algorithms and reducing the instability associated with reliance on a single predictor. In cryptocurrency forecasting, ensemble approaches have demonstrated considerable potential for extracting complex relationships from highly volatile time-series data [2]. Similarly, deep-learning architectures have been developed for high-frequency, short-term cryptocurrency prediction because they can learn hierarchical and nonlinear representations from large volumes of rapidly changing market information [3]. Machine learning has additionally been integrated with risk-adjusted portfolio optimization, thereby linking price forecasting with asset-allocation decisions and emphasizing that predictive accuracy should ultimately be evaluated in relation to financial risk and decision quality [4]. Nevertheless, although data-driven methods can produce accurate empirical forecasts, they often provide limited insight into the continuous-time dynamics, probabilistic structure, and mathematical mechanisms underlying price variation. This limitation sustains the importance of stochastic differential-equation models, which provide an interpretable mathematical framework for representing drift, volatility, random shocks, and jump behavior.

Stochastic differential equations extend ordinary differential equations by incorporating one or more stochastic terms into the system dynamics. In a general Itô stochastic differential equation, the drift coefficient represents the systematic or expected direction of change, whereas the diffusion coefficient measures the intensity of random variation. The stochastic component is commonly represented by Brownian motion or a more general stochastic process. The mathematical foundations of this framework are provided by stochastic calculus, which develops the concepts of stochastic processes, filtrations, Itô integrals, stochastic differentials, and probabilistic solution properties [5]. From a formal perspective, stochastic differential equations are particularly useful when the evolution of a variable cannot be represented by a smooth deterministic trajectory because the system is continuously exposed to random influences. Øksendal provided a rigorous yet accessible formulation of Itô calculus, Brownian motion, Itô's formula, diffusion processes, and their applications, establishing the principal theoretical tools required for constructing and analyzing stochastic models in finance and other applied sciences [6]. These models allow researchers to describe an ensemble of possible future trajectories rather than generate only one deterministic forecast. Consequently, they can be used to estimate expected values, variances, probability distributions, confidence intervals, and risk measures associated with future prices.

The application of stochastic differential equations is not limited to financial markets. Their broad relevance stems from the fact that many biological, physical, engineering, and economic systems are influenced by uncertain disturbances. Numerical simulations of stochastic ordinary differential equations in biomathematical models have shown that stochastic formulations can represent variability that deterministic equations overlook, particularly when biological systems are affected by environmental or demographic noise [7]. In electrical engineering, stochastic modeling of RL circuits under different noise structures has likewise demonstrated that the choice of stochastic perturbation substantially affects the estimated system response. Farnoosh and colleagues examined an RL electrical circuit by introducing alternative noise terms and showed that a model incorporating combined stochastic influences could more accurately describe system behavior than a model restricted to a single conventional noise source [8]. These applications illustrate a general methodological principle: model accuracy depends not merely on including a random term, but on selecting a stochastic process whose distributional and temporal properties correspond to the actual disturbances affecting the system.

In financial modeling, one of the most influential stochastic differential-equation frameworks is the Black–Scholes model. Although originally developed for option pricing, the underlying geometric Brownian motion formulation has become a fundamental model for describing the evolution of stock prices, exchange rates, commodities, and other financial assets. In its basic form, the model assumes that the relative change in an asset price consists of a deterministic drift component and a stochastic diffusion component driven by Brownian motion. This structure implies that continuously compounded returns are normally distributed and that asset prices follow a lognormal distribution. The model is analytically convenient, economically interpretable, and compatible with a range of estimation and simulation methods. However, its standard assumptions may be restrictive in markets characterized by skewness, heavy tails, volatility clustering, structural breaks, and sudden jumps. The Iranian currency market provides a particularly relevant setting for evaluating these limitations because observed exchange-rate series may display rapid upward movements, discontinuities, and irregular fluctuations that cannot be plausibly represented as small, continuous Gaussian disturbances alone.

A major advantage of stochastic differential equations is that they can be modified by replacing the standard Brownian driver with alternative noise processes. Gaussian white noise is the most common specification because it is mathematically tractable and directly connected to the Wiener process. In a Brownian framework, increments over non-overlapping intervals are independent and normally distributed, while sample paths are continuous. These assumptions are suitable for modeling the accumulation of many small random shocks, but they are less appropriate when the observed series contains abrupt movements. In financial markets, large exchange-rate changes may occur following political announcements, unexpected macroeconomic data, military conflict, sanctions, regulatory decisions, or sudden changes in market expectations. Such movements may produce return distributions with excess kurtosis, asymmetry, and discontinuities. Therefore, the stochastic process selected for the model must reflect whether price variation is primarily continuous or whether it includes jump behavior.

Mixed noise constitutes one possible extension of the Gaussian formulation. It may be defined as a weighted linear combination of two or more independent Wiener processes. This structure allows the model to represent several simultaneous sources of Gaussian uncertainty rather than treating market randomness as a single homogeneous disturbance. The use of mixed noise can be justified when different economic, political, institutional, and behavioral factors contribute to exchange-rate fluctuations. Because the component processes may be assigned different weights, mixed-noise models can represent the combined contribution of distinct stochastic influences. Earlier research has shown that replacing a single white-noise term with mixed noise may improve the correspondence between stochastic simulations and observed system behavior. In the context of stochastic population models, confidence intervals were developed for exponential growth processes incorporating mixture noise, demonstrating how alternative stochastic structures influence interval estimation and uncertainty assessment [9]. More generally, the theory and applied formulation of stochastic differential equations provide the mathematical basis for defining mixed-noise processes and incorporating them into numerical models [10].

Although mixed noise increases the flexibility of Gaussian models, a linear combination of Wiener processes remains Gaussian under standard conditions. Consequently, mixed noise may capture multiple continuous sources of uncertainty but cannot, by itself, fully represent discontinuous jumps or heavy-tailed shocks. This distinction is important because the empirical improvement obtained by using mixed noise does not necessarily imply that the underlying market dynamics are Gaussian. A model may become more accurate because it represents several volatility sources, yet still underestimate the probability and magnitude of extreme price changes. For markets in which discontinuities are economically meaningful, a more general stochastic process is needed.

Lévy processes offer such a generalization. A Lévy process has stationary and independent increments and is stochastically continuous, but unlike Brownian motion, it may include a jump component. Through the Lévy–Khintchine representation, a Lévy process can be decomposed into deterministic drift, Brownian variation, and jumps described by a Lévy measure. This flexibility enables Lévy-driven models to represent continuous fluctuations, occasional large shocks, asymmetric movements, and heavy-tailed distributions. Such properties are especially relevant to financial-price series, where extreme returns occur more frequently than predicted by the normal distribution. Modeling financial variables with Lévy noise therefore provides a theoretically coherent way to accommodate both regular market variation and abrupt price changes.

Recent methodological developments have improved the simulation of complex Lévy processes. Godsill and Kindap proposed point-process methods for simulating generalized inverse Gaussian processes and estimating the associated Jaeger integral, contributing to the computational treatment of non-Gaussian stochastic processes [11]. Their work is important because generalized inverse Gaussian processes are related to flexible classes of jump models that can represent asymmetric and heavy-tailed behavior. Kindap and Godsill subsequently developed point-process simulation procedures for generalized hyperbolic Lévy processes, further extending the computational tools available for simulating non-Gaussian processes used in statistics and finance [12]. Generalized hyperbolic distributions are particularly valuable in financial modeling because they can reproduce empirical features such as skewness and excess kurtosis more effectively than Gaussian distributions. These advances strengthen the feasibility of employing Lévy-based processes in applied prediction problems.

Applications of Lévy noise to financial markets have also provided evidence that jump-based stochastic models may outperform purely Gaussian specifications. Mohammadi and Nabati modeled financial markets using a combined Ornstein–Uhlenbeck process driven by Lévy noise and showed that incorporating non-Gaussian disturbances could produce estimates that remained stable around observed market values [13]. The Ornstein–Uhlenbeck process is particularly suitable for variables exhibiting mean reversion, while the addition of Lévy noise allows the model to accommodate discontinuous shocks. Their findings support the broader proposition that the stochastic driver should be selected according to the empirical characteristics of the financial series. When price data contain abrupt shifts, a continuous Brownian specification may generate substantial forecast error because it distributes uncertainty smoothly over time instead of representing discrete jumps.

Stochastic differential equations have previously been used to simulate and forecast commodity prices. Farnoosh and colleagues applied stochastic differential equations to the simulation and forecasting of OPEC oil prices, illustrating the relevance of stochastic modeling to markets exposed to economic, political, and supply-related uncertainty [14]. Oil and foreign-exchange markets share several modeling challenges, including sensitivity to geopolitical events, policy changes, speculative expectations, and global economic conditions. Both may exhibit non-stationarity, sudden price adjustments, and periods of elevated volatility. Accordingly, the methodological logic used in stochastic commodity-price forecasting can be extended to exchange-rate prediction, provided that model parameters and noise structures are adapted to the characteristics of the currency series.

The practical implementation of stochastic differential equations usually requires numerical approximation because closed-form solutions are unavailable for many models, particularly when coefficients are nonlinear or the stochastic driver is non-Gaussian. Kloeden and Platen developed a comprehensive theory of numerical solutions for stochastic differential equations, including strong and weak approximation, stochastic Taylor expansions, convergence analysis, and simulation algorithms [15]. Among the most widely used procedures is the Euler–Maruyama method, which is the stochastic analogue of the deterministic Euler method. For an Itô equation, the procedure approximates the next value of the process by adding the drift increment and a stochastic increment determined by the diffusion coefficient and the simulated noise. The method is computationally straightforward and therefore suitable for comparative simulations involving several noise types.

Despite its simplicity, the Euler–Maruyama method must be used with attention to convergence conditions, discretization intervals, and the properties of the driving process. Under standard Lipschitz and linear-growth assumptions, it has strong convergence of order one-half for Brownian-driven equations, while its weak convergence may be of order one under sufficient smoothness conditions. When Lévy noise is introduced, the simulation algorithm must additionally account for jump increments, their frequency, and their distribution. Therefore, comparative forecasting should not be interpreted merely as replacing one random-number generator with another. Each noise structure represents a different probabilistic hypothesis about how uncertainty enters the price process. White noise assumes continuous Gaussian variation, mixed noise represents several combined Gaussian influences, and Lévy noise allows both continuous fluctuations and discontinuous jumps.

Another important issue is parameter estimation. In stochastic financial models, the drift coefficient describes the expected proportional rate of change, while the diffusion coefficient reflects volatility. These parameters may be estimated through maximum likelihood, moment-based estimators, or discretized approximations derived from observed returns. Reliable estimation is essential because simulation accuracy depends jointly on the model structure and the parameter values. Even a theoretically appropriate noise process may generate poor forecasts if drift and volatility are estimated inaccurately. Conversely, a simpler model may perform reasonably well over a short horizon when its parameters are closely calibrated to recent observations. For this reason, the comparative evaluation of noise types should use the same empirical series, estimation framework, numerical method, and forecast horizon so that differences in prediction error can be attributed primarily to the stochastic specification.

Prediction intervals are also central to the assessment of stochastic models. A point forecast provides only one estimated future price, whereas a confidence or prediction interval communicates the degree of uncertainty surrounding that estimate. In volatile currency markets, interval estimation is especially important because the practical consequences of forecast error may be substantial for investors, importers, exporters, policymakers, and households. Stochastic models naturally support interval construction through the estimated mean and variance of the future price distribution. The quality of an interval should be assessed according to both coverage and width: an interval that is too narrow may fail to contain the realized price, while an excessively wide interval may provide little useful information. The integration of parameter estimation, numerical simulation, point prediction, and interval estimation therefore offers a more comprehensive approach to currency-price forecasting than reliance on a single deterministic projection.

The comparison of white, mixed, and Lévy noise is particularly meaningful in the Iranian exchange-rate context because each specification corresponds to a distinct interpretation of market uncertainty. A white-noise model suggests that exchange-rate changes arise from many small and approximately Gaussian shocks. A mixed-noise model suggests that several independent sources of continuous uncertainty operate simultaneously. A Lévy-noise model assumes that regular fluctuations coexist with occasional large shocks that generate jumps. Evaluating these competing formulations against observed exchange-rate values can therefore provide both predictive and substantive insight. A lower forecasting error under Lévy noise would indicate that discontinuities are an important feature of the price process, whereas superior performance under mixed noise would suggest that multiple continuous Gaussian disturbances are sufficient to explain the observed dynamics.

Accordingly, the aim of this study is to numerically compare the predictive efficiency of stochastic Black–Scholes models driven by Gaussian white noise, mixed noise, and Lévy noise for forecasting the currency price in Iran using the Euler–Maruyama method.

2. Methodology

Stochastic calculus pertains to the study of stochastic processes, stochastic integrals, and stochastic differential equations. To study these topics, one needs to delve into the theories of probability and stochastic processes.

Random Variable:

Suppose (Ω, F, P) is a probability space. We call X : Ω →R a random variable on the probability space (Ω, F, P).

Stochastic Process

In many physical applications, a sequence of random variables can often be considered a simple description of a probabilistic system at discrete moments in time. Thus, the behavior of such a system can be described by a family of random variables leading to a stochastic process. A stochastic process is a family of random variables X(t,ω) with two variables t∈T and ω∈Ω, on the shared space (Ω, F, P), where the values are assumed to be real and are considered as a function of ωω for each fixed t. The parameter t is interpreted as time in the time set TT. For each fixed t, X(t,.) is a random variable on the probability space, and for each fixed ω, X(.,ω) is called a sample path or trajectory of the stochastic process. If the time set TT is assumed to be discrete, then X(t,ω) is considered a discrete-time stochastic process; whereas if the time set TT is a bounded or unbounded interval, then X(t,ω) is considered a continuous-time stochastic process. Intuitively, if t is assumed to be time and ω a particle (or experiment), then X(t,ω) can be interpreted as representing the position (or result) of a particle (or experiment) at time t. For each fixed ω, the stochastic process X(t,ω) is denoted by X(t)X(t), so in this paper, whenever we refer to the stochastic process X(t)X(t), we mean the stochastic process X(t)X(t) for a fixed ω.

Wiener Process

A stochastic process $W(t)$ ،$t \in [0,\infty)$،We call the stochastic process W(t)W(t), t∈[0,∞), a Brownian motion process (Wiener process) if it satisfies the following conditions:

a. The paths of W(t) are almost surely (a.s) continuous.

b. P(W(0)=0) = 1.

c. For $0 < t_0 < t_1 < \cdots < t_n$، the increments

$$W(t_1) - W(t_0), W(t_2) - W(t_1), \cdots, W(t_n) - W(t_{n-1})$$

are independent.

d. If t and h>0 are fixed positive values, then W(t+h)−W(t) has a Gaussian distribution with mean zero and variance h.

This means that for every $0 \leq s \leq t$ ، the distribution of W(t)−W(s) has a normal distribution with mean zero and variance t−s. In other words, the increments $W(t_1) - W(t_0), W(t_2) - W(t_1), \cdots, W(t_n) - W(t_{n-1})$have stationary increments (they are stationary increments).

Several Important Properties of the Wiener Process:

a. The Wiener process has continuous sample paths.

b. Brownian motion has no derivatives at almost any point along its path.

c. Brownian motion possesses the Markov property.

d. Brownian motion has the martingale property.

e. The Brownian path is nowhere monotone in any interval.

Ito's Formula (Ito's Chain Rule) in One Dimension

Suppose X(t) is an Ito process, meaning:

$$X(t) = X(0) + \int_0^t u(s,\omega)ds + \int_0^t v(s,\omega)dB_{s,}$$

$$dX(t) = udt + vdB_t,$$

Where

$$P[\int_0^t v^2(s,\omega)ds < \infty, \qquad for\ all\ \ t \geq 0] = 1,$$

$$P[\int_0^t |u(s,\omega)|\ ds < \infty, \qquad for\ all\ t \geq 0] = 1.$$

and $g(t,x) \in C^2([0,\infty) \times R)$، then $Y(t) = g(t, X(t))$ Once again, an Ito process is defined as:

$$dY(t) = \frac{\partial g}{\partial t}(t, X(t))dt + \frac{\partial g}{\partial x}(t, X(t))dX(t) + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t, X(t))\big(dX(t)\big)^2,$$

where

$$(dX(t))^2 = dX(t).dX(t), \quad dt.dt = dt.dX(t) = dX(t).dt = 0, dB_t.dB_t = dt.$$

Ito Integral

Suppose β is a Borel sigma-algebra on [0, ∞) ،F is a sigma-algebra on Ω, and ν=ν(S,T) is a function from the class

$$f(t,\omega): [0,\infty) \times \Omega \to R$$

In this case f(t, ω) →(t, ω) is measurable with respect to β×F, and

$$E\ [\int_S^T f^2(t,\omega)dt] < \infty.$$

Definition:

Suppose f∈ ν(S,T). The Ito integral of f from SS to T is defined as follows:

$$\int_S^T f(t,\omega)dB_t(\omega) = Lim_{n\to\infty}\int_S^T \varphi_n(t,\omega)dB_t(\omega).$$

When ϕn\phi_n is a sequence of functions that satisfy the following condition:

$$E\ [(\int_S^T (f(t,\omega) - \varphi_n(t,\omega))^2\ dt\ ] \to 0, \qquad n \to \infty.$$

Properties of the Ito Integral:

Suppose $F, g \in V(0\ ,T)$ and $0 \leq S \leq U \leq T$, then:

1) $\int_S^T f dB_t = \int_S^U f dB_t + \int_U^T f dB_t$

۲) $\int_S^T (cf+g)dB_t = c\int_S^T f dB_t + \int_S^T g dB_t$

۳) $E\left(\int_S^T dB_t\right) = 0$

Proof: Refer to page 30 of Øksendal [10].

1-5-3 Itô isometry properties

Suppose f∈ ν(S,T) In this case (for proof, refer to page 29 of Øksendal [10]):

$$E\left[\left(\int_S^T f(t,\omega)dB_t(\omega)\right)^2\right] = E\left[\int_S^T f^2(t,\omega)dt\right]$$

Stochastic Differential Equation

Any relation between an independent variable, a dependent variable, and their derivatives is called an ordinary differential equation (ODE). By adding a stochastic term to the ODE, a stochastic differential equation (SDE) is formed. Therefore, each SDE is a differential equation where one or more terms are stochastic processes. This causes the solutions of the differential equation to become stochastic processes. In general, an SDE can be considered an equation provided along with a random variable (such as white noise). Thus, stochastic differential equations are useful in describing Brownian motion or the Wiener process.

$$dX(t) = f\big(t, X(t)\big)dt + g\big(t, X(t)\big)dW(t), t \in [t_0, T], T > 0,$$

where f(t,X(t)) is the drift coefficient, g(t,X(t)) is the diffusion coefficient, and W(t) is the Brownian motion.

The integral form of the above stochastic differential equation is as follows:

$$X(t) = X(0) + \int_{t_0}^t f(s, X(s))ds + \int_{t_0}^t g\big(s, X(s)\big)dW(s), \quad where\ t \in [t_0, T]$$

In this equation, the first integral on the right-hand side of equation (2-2) is a Riemann integral, and the second integral is a stochastic integral.

The Black-Scholes Stochastic Differential Equation [11]

Assume P(t) is the value of a stock at time t. $\frac{dP}{P}$ represents the rate of change in stock prices, and P(0) is the price of a stock at the beginning. The goal is to calculate $P(t_n)$ i.e., the price at nn future time steps, and the general stochastic model related to the value of the stock at any moment, which is $\frac{dP}{P}$

Suppose $t_0 < t_1 < \cdots < t_{n-1} < t_n$are equally spaced points; in this case, we have:

$$P(t_n) = P(t_0).\frac{P(t_1)}{P(t_0)}.\cdots.\frac{P(t_n)}{P(t_{n-1})}$$

The ratios $R_k = \frac{P(t_k)}{P(t_{k-1})}$represent the growth of the stock price in financial markets in the ideal case over the time interval $[t_{k-1}, t_k]$. It can be assumed that these numbers are log-normal random variables, meaning they have the following probability density function:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}x} e^{\frac{-(Lnx-\rho)^2}{2\sigma^2}}$$

Consequently, we have:

$$\ln P(t_n) = \ln P(t_0) + \sum_{k=1}^{n} \ln \frac{P(t_k)}{P(t_{k-1})}$$

where

$$\sum_{k=1}^{n} Ln\frac{P(t_k)}{P(t_{k-1})}$$

Is the sum of normal variables with mean ρ and variance σ^2. Therefore:

$$Ln\, P(t_n) - LnP(t_0) = N(n\rho, n\sigma^2) = nN(\rho, \sigma^2),$$

$$Ln\frac{P(t_n)}{P(t_0)} = n\, N(\rho, \sigma^2) = n\big(\rho + N(0, \sigma^2)\big) = n\big(\rho + \sigma N(0,1)\big)$$

$$\Rightarrow \frac{P(t_n)}{P(t_0)} = e^{n(\rho+\sigma N(0,1))}$$

$$\Rightarrow \frac{P(t)}{P(t_0)} = e^{t(\rho+\sigma N(0,1))}$$

$$\Rightarrow P(t) = P(t_0)e^{\rho t+\sigma W(t)}$$

we call the above relationship the Black-Scholes-Merton formula related to the value of the stock, $N(0,t) = \sqrt{t}N(0,1)$.

The differential form of this formula is:

$$\frac{dP}{P} = \rho dt + \sigma dW_t$$

where ϱ is a positive constant drift coefficient, and σ is the diffusion coefficient. Thus:

$$dP = \rho P dt + \sigma P dW_t. \tag{1-3}$$

Using Ito's formula, we have:

$$d(LnP) = \frac{dP}{P} - \frac{\sigma^2}{2}\frac{P^2}{P^2}dt = \left(\rho - \frac{\sigma^2}{2}\right)dt + \sigma dW_t$$

then

$$P(t) = P_0 e^{\left(\rho-\frac{\sigma^2}{2}\right)t+\sigma W_t}$$

From (1-3), we have:

$$P(t) = P_0 + \int_0^t \rho P dS + \int_0^t \sigma P dW_t$$

Since $E\left(\int_0^t \sigma P dW_t\right) = 0$، (see Kloeden and Platen, 2000), we have:

$$E\big(P(t)\big) = P_0 + \int_0^t \rho\, E(P) ds$$

Thus:

$$E\big(P(t)\big) = P_0 e^{\left(\rho-\frac{\sigma^2}{2}\right)t}, \qquad t > 0.$$

As a result, the expected value of a stock price at any time corresponds to the actual solution of the ordinary differential equation for σ=0. Given the model is a stochastic differential equation, we aim to estimate its parameters σ\sigma and ϱ. Without loss of generality, assume the time points are equidistant. With this assumption, the continuous SDE can be discretized as follows:

$$P(t+\Delta t) - P(t) = \rho P(t)\Delta t + \sigma X(t)(W(t+\Delta t) - W(t))$$

$$\frac{P(t+\Delta t) - P(t)}{P(t)\Delta t} = \rho + \frac{\sigma}{\Delta t}(W(t+\Delta t) - W(t))$$

By taking the expectation on both sides of the above equation, noting W(t+Δt)−W(t) has a normal distribution with zero mean and variance Δt, we have:

$$E\left(\frac{P(t+\Delta t) - P(t)}{P(t)\Delta t}\right) = \rho.$$

Therefore, for the equally spaced time data in the table above, the estimate for the drift parameter ϱ is:

$$\hat{\rho} = \frac{1}{N}\sum_{i=1}^{n}\frac{P(t_i) - P(t_{i-1})}{\Delta t\, P(t_i)}, \qquad t_i = t_{i-1} + it.$$

To estimate the diffusion coefficient σ:

$$\frac{P(t+\Delta t) - P(t)}{\Delta t} - \rho P(t) = \sigma P(t)\frac{W(t+\Delta t) - W(t)}{\Delta t}$$

Knowing $(W(t+\Delta t) - W(t))^2 = \Delta t$. we square both sides and take expectations:

$$(P(t+\Delta t) - P(t) - \rho P(t)\Delta t)^2 = \sigma^2 P^2(t)(\Delta W(t))^2$$

$$E\left[\frac{1}{\Delta t}\left(\frac{P(t+\Delta t) - P(t)}{P(t)} - \hat{\rho}\Delta t\right)^2\right] = \sigma^2$$

Using this idea, the diffusion coefficient σ\sigma for the discretized SDE with equally spaced time points is estimated as follows:

$$\hat{\sigma}^2 = \frac{1}{\Delta t}\frac{1}{N}\sum_{i=1}^{n}\left(\frac{P(t_i) - P(t_{i-1})}{P(t_i)} - \hat{\rho}\Delta t\right)^2.$$

(For more details on estimating parameters of stochastic models, refer to sources [8] and [11]).

To obtain a 95% confidence interval for P(t), first find the mean and variance of the solution to the Black-Scholes SDE (3-1) [12]:

$$P(t) = P_0 e^{\left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t} \Rightarrow Ln\big(P(t)\big) = Ln(P_0) + \left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t$$

Thus:

$E\left(Ln\big(P(t)\big)\right) = Ln(P_0) + \left(\rho - \frac{\sigma^2}{2}\right)t,$

$$Var\left(Ln\big(P(t)\big)\right) = \sigma^2 t$$

Therefore:

$E\big(P(t)\big) = exp[\, Ln(P_0) + \rho t] = P_0 e^{\rho t},$

$$Var(P(t)\,) = \exp\left[2Ln(P_0) + 2\left(\rho - \frac{\sigma^2}{2}\right)t\right]e^{\sigma^2 t}[e^{\sigma^2 t} - 1]$$

Hence, a 95% confidence interval for P(t) is:

$$P(t) \in \big(a(t) - Z_{0.025}.\, b(t), a(t) + Z_{0.025}.\, b(t)\big)$$

or

$$P(t) \in \big(a(t) - 1.96b(t), a(t) + 1.96\, b(t)\big)$$

where:

$a(t) = P_0 e^{\rho t},\ b(t) = exp\left[2Ln(P_0) + 2\left(\rho - \frac{\sigma^2}{2}\right)t\right]e^{\sigma^2 t}[e^{\sigma^2 t} - 1].$

(See the paper by Rezaeyan and Jafari at the 46th Mathematics Conference in Yazd [12]).

Mixed Noise

Mixed noise is a linear combination of Wiener processes. We call the stochastic process M(t) a mixed noise if it satisfies the following linear collective stochastic differential equation [12, 4]:

$$dM(t) = \sum_{k=1}^{n}\alpha_k dW_k(t), \qquad \sum_{k=1}^{n}\alpha_k = 1$$

where W_i are independent Wiener processes and α_i are constant values.

Without loss of generality, for simplicity and to verify the claims of this paper, we consider the mixture noise as follows:

$$dM_t = \alpha_1 dW_1(t) + \alpha_2 dW_2(t) + \cdots + \alpha_k dW_k(t)$$

with $\alpha_i = \frac{1}{k}$, $i = 1, 2, \cdots, k$.

Therefore, the Black-Scholes model with mixed noise is as follows:

$$dP_t = \rho P_t dt + \sigma P_t d\mathrm{M}_\mathrm{t}.$$

$$dP_t = \rho P_t dt + \frac{\sigma}{k} P_t \big(dW_1(t) + dW_2(t) + \cdots + dW_k(t)\big)$$

The parameters μ and σ are estimated based on the actual values as follows:

$$\hat{\rho} = \frac{1}{N} \sum_{i=1}^{n} \frac{P(t_i) - P(t_{i-1})}{\Delta t \, P(t_i)}, \qquad t_i = t_{i-1} + it.$$

$$\hat{\sigma}^2 = \frac{1}{\Delta t} \frac{1}{N} \sum_{i=1}^{n} (\frac{P(t_i) - P(t_{i-1})}{P(t_i)} - \hat{\rho} \Delta t)^2$$

Mixture noise comprises a combination of multiple noises with different distributions, which can better model large and unexpected fluctuations in the market. These noises, due to their varying combinations, can reflect more information from real market data. Also, using mixture noise in stochastic models can increase the accuracy of predictions as these noises have a greater ability to adapt to complex realities and different variables.

Lévy Process

As mentioned in the introduction, Gaussian noise does not support market jumps. Therefore, to model more realistically, we should use noise that includes jumps. Such noise is called Lévy noise. Due to its discontinuous characteristics, Lévy noise can simulate sudden and significant market changes that Gaussian white noise cannot. This feature is particularly useful in financial markets that experience sudden jumps and drops. Lévy noise, because it is generated by infinitely divisible distributions, can model a wider range of random changes in financial data. A stochastic process on R^n like $\{X_t\}_{t\epsilon[0,T]}$ is called a Lévy process (Lévy noise) if it satisfies the following conditions:

a. For any $n > 0$ and $t_0 < t_1 < \cdots < t_n$, the process has independent increments, meaning the following random variables are independent of each other:

b. $X_0 = 0$

c. The distribution of $X_{t+s} - X_s$ does not depend on s, meaning the process has stationary increments.

d. The process is stochastically continuous.

e. For each ω∈Ω, $X_t(\omega)$ is right-continuous for $t \leq 0$ and has a left limit for $t > 0$.

Infinitely Divisible Distribution:

A random variable X is called infinitely divisible if, for any n=1,2,… there exist random variables ${X_1}^{(n)}, {X_2}^{(n)}, \dots, {X_n}^{(n)}$ that are identically distributed and independent of each other such that ${X_1}^{(n)} + {X_2}^{(n)} + \cdots + {X_n}^{(n)}$has the same distribution as X (Gregory, 2002).

Theorem:

Suppose $\{X_t ؛ t \geq 0\}$ is a Lévy process. In this case, for each t≥0 the process $\{X_t\}_{t\geq0}$has an infinitely divisible distribution F. Conversely, if F is an infinitely divisible distribution, then a Lévy process $\{X_t\}_{t\geq0}$exists such that the distribution of $X_1$is given by F (Gregory, 2002).

From Lévy decomposition, we observe that, in general, each Lévy process consists of three independent parts: a deterministic function, a Brownian component, and a pure jump component. A Lévy process is considered a non-Gaussian process (Barndorff and Shephard, 2000).

Euler-Maruyama Method for Numerical Solution of SDEs

One of the simplest numerical approximations for SDEs is the Euler-Maruyama method. When only the first term in the stochastic Ito-Taylor series expansion is considered and the rest are ignored, the Euler or Euler-Maruyama method is obtained:

$$X(t_{i+1}) = X(t_i) + f\big(X(t_i)\big)\Delta t + g\big(X(t_i)\big)\Delta W_i$$

for i=0,1,…,N−1 with the initial value $X(t_0) = x_0$. The Euler-Maruyama approximation is strongly convergent of order 0.5 if the coefficients f and g satisfy the Lipschitz conditions and bounded growth conditions. Milstein (1979) and Talay (1984) showed that the Euler-Maruyama approximation is weakly convergent of order one if the coefficients are sufficiently smooth [8]. Note that the order of weak convergence is greater than that of strong convergence.

3. Findings and Results

To predict the exchange rate (dollar) in Iran, we considered its actual price in Rials from the website for the period from the 3rd of Shahrivar 1403 to the 22nd of Azar 1403 (working days). The values are provided in Appendix A. First, using the Maple software, we plotted the price versus time graph, resulting in Figure 8-1.

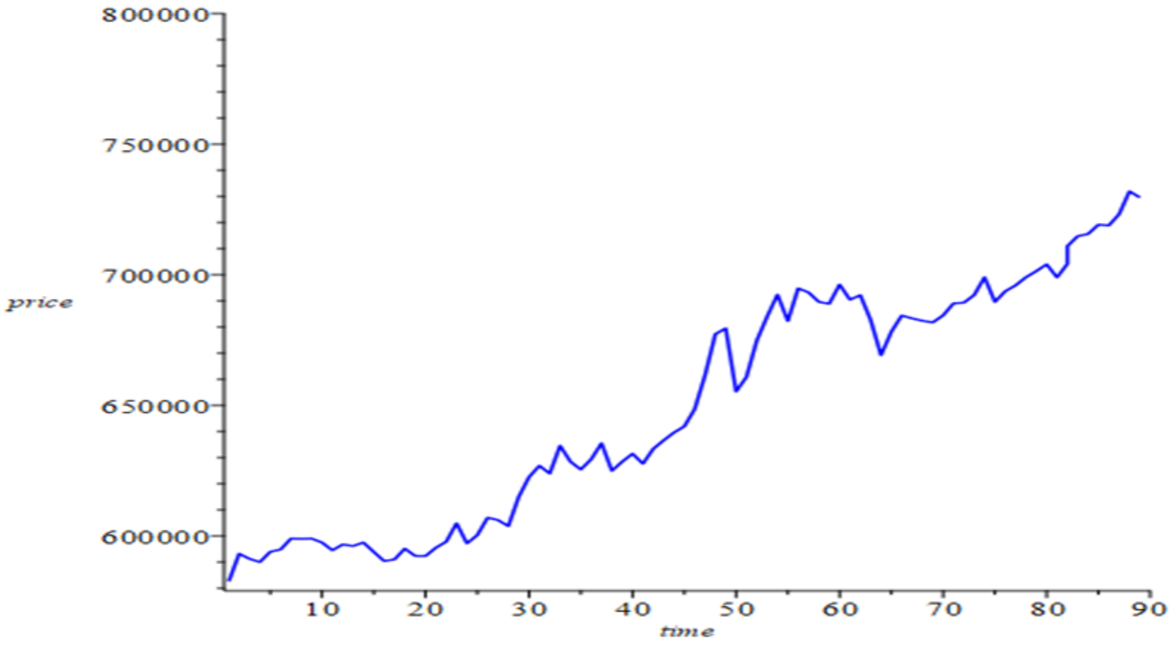

Figure 1: Real Price Chart of the Dollar in Iran from Shahrivar 3rd, 1403 to Azar 22nd, 1403

Using equations from previous section with the Maximum Likelihood Estimation (MLE) method, we estimated the model parameters based on the observations in Appendix A using the Maple software. The estimated values are as follows:

$$\rho = 0.0467$$
$$\sigma = 0.0409$$

These estimates provide a foundation for modeling and predicting the future behavior of the dollar price in Iran.

A 95% confidence interval for predicting the dollar price in Iran, based on the Black-Scholes model and observations in Appendix A, is obtained as follows:

P(t)∈(a(t)−1.96·b(t), a(t)+1.96·b(t))

where:

$a(t) = P_0 e^{\rho t},\ b(t) = exp\left[2Ln(P_0) + 2\left(\rho - \frac{\sigma^2}{2}\right)t\right] e^{\sigma^2 t}\left[e^{\sigma^2 t} - 1\right].$

This interval provides a range within which we can be 95% confident that the actual future price of the dollar will fall. A 95% confidence interval for predicting the dollar price in Iran based on the Black-Scholes model, considering the observations in Appendix A, is (717,877.46405376, 745,014.70867383). This means that in the worst-case scenario, the dollar price in Iran on December 13, 2024, is approximately 718,000 Rials, and in the best-case scenario, the dollar price on that day is 745,000 Rials. The actual dollar price on December 13, 2024, was 734,700 Rials, which indicates the accuracy and validity of the interval estimation of the dollar price in Iran based on the proposed model.

In this step, we simulated the Black-Scholes equation using the Euler-Maruyama numerical method based on the daily observations of the dollar price in Iran (in Rials) from the 3rd of Shahrivar of the current year to the 22nd of Azar 1403, with three types of noise: Gaussian white noise, mixed noise, and Lévy noise. The following graphs were obtained (the Maple program related to each simulated graph is provided in Appendix B):

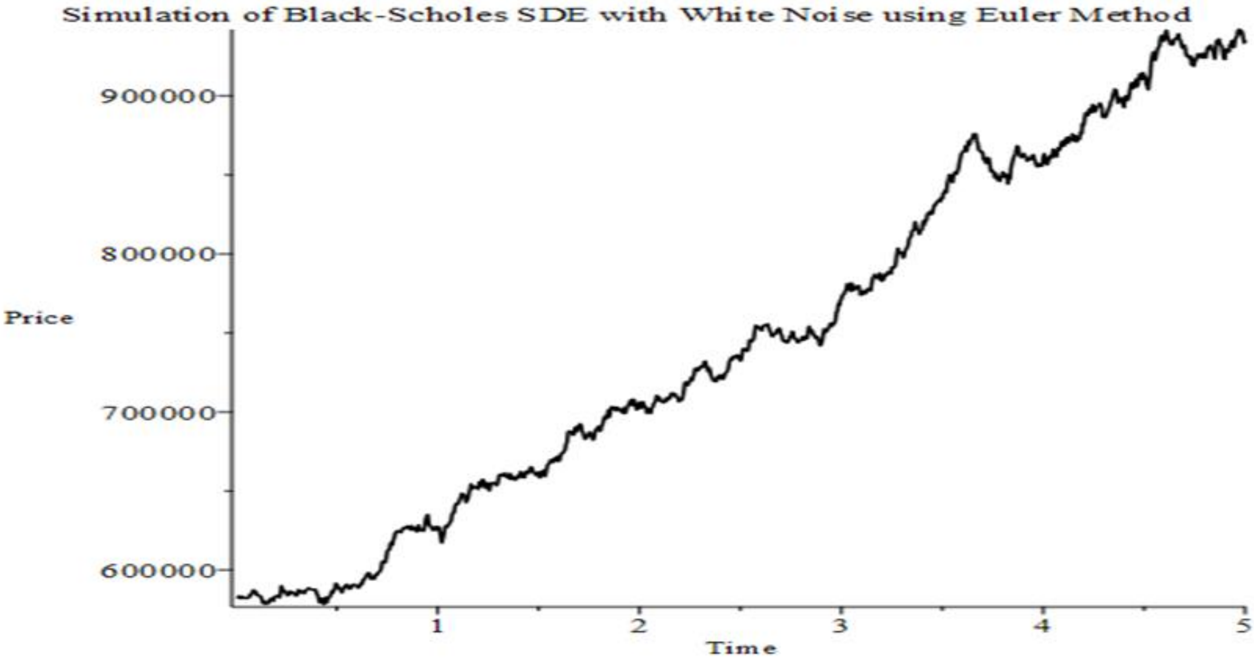

Figure 2: Simulated Dollar Price in Iran using the Black-Scholes Equation with Gaussian White Noise through the Euler-Maruyama Numerical Method

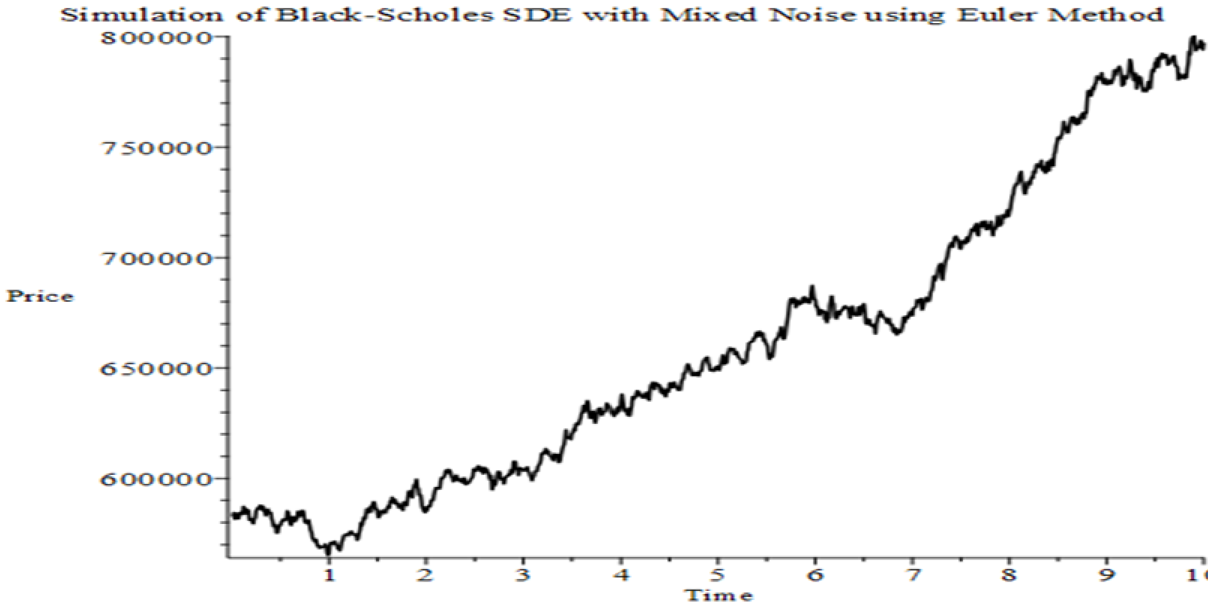

Figure 3: Simulated Dollar Price in Iran using the Black-Scholes Equation with Mixed Noise through the Euler-Maruyama Numerical Method

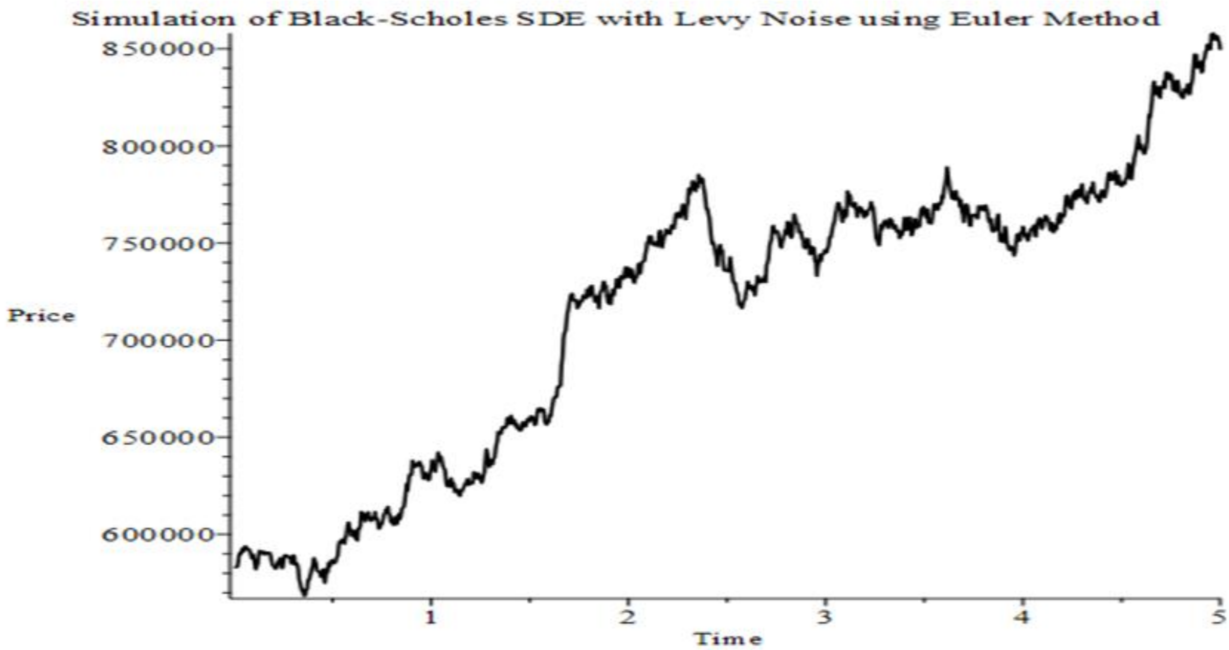

Figure 4: Simulated Dollar Price in Iran using the Black-Scholes Equation with Levy Noise through the Euler-Maruyama Numerical Method

To evaluate the accuracy of the Black-Scholes model using the Euler-Maruyama numerical method for predicting the dollar price in Iran, we replaced the Gaussian white noise with mixed noise and Lévy noise. We then predicted the dollar price ten days after the 22nd of Azar 1403, which is the 2nd of Dey 1403, P^(100)\hat{P}(100), using the three previously mentioned graphs and compared it with the actual value P(100)P(100). We calculated the error for each scenario. The results are presented in Table (7-1) below (the actual dollar price in Iran on the 2nd of Dey 1403 is P(100)=762800 Rials):

Table 1. Comparison of Predicted Dollar Prices in Iran using the Euler-Maruyama Numerical Method with the Black-Scholes Equation, employing Mixed and Lévy Noise instead of Gaussian White Noise

Noise Type	Actual Price, $\boldsymbol{P}(\mathbf{100})$	Predicted Price, $\widehat{\boldsymbol{P}}(\mathbf{100})$	$\boldsymbol{E}$rror $\widehat{\boldsymbol{e}}(\mathbf{100}) = \lvert \boldsymbol{P}(\mathbf{100}) - \widehat{\boldsymbol{P}}(\mathbf{100}) \rvert$
Gaussian White	762800	937093	174293
Mixed Noise	762800	794487	31687
Levy Noise	762800	751867	10933

From Table 1, we observe the error in predicting the dollar value in Iran (in Rials) based on the Black-Scholes equation using the Euler numerical method. It is clear that the Black-Scholes model with mixed noise is more efficient than with Gaussian white noise. However, considering the dollar price in Iran over recent months, influenced by the situation in the Middle East, showing jumps, indicates that the noise distribution in the Black-Scholes model cannot be Gaussian. This means the process used in the Black-Scholes model cannot be a Wiener process (or Brownian motion), as the sample observations in Appendix A exhibit jumps.In such cases, instead of a Gaussian process, we must use a Lévy process. The Lévy process, in addition to possessing the properties of a Gaussian process, also accounts for jumps. The simulation results of the Black-Scholes model confirm this, as the prediction error with Lévy noise instead of Gaussian white noise in the Black-Scholes model is significantly lower. Furthermore, using Lévy noise instead of mixed noise in this method proves to be more efficient, as the Black-Scholes model with Lévy noise has less error compared to mixed noise. Therefore, the Black-Scholes model with Lévy noise is more efficient than white and mixed noise, which is a linear combination of several Gaussian white noises. In conclusion, when modeling stock and currency values based on initial observations, if jumps are observed in the initial observations, the most suitable and efficient model for prediction is one that uses Lévy noise.

4. Discussion and Conclusion

The present study compared the predictive efficiency of the stochastic Black–Scholes model under three alternative stochastic specifications—Gaussian white noise, mixed noise, and Lévy noise—for forecasting the dollar exchange rate in Iran. The models were solved numerically through the Euler–Maruyama method, while the drift and diffusion parameters were estimated from observed daily exchange-rate data. The results demonstrated substantial differences in predictive performance according to the type of noise incorporated into the model. The actual dollar price at the target prediction point was 762,800 rials, whereas the Black–Scholes model with Gaussian white noise predicted a value of 937,093 rials, resulting in an absolute prediction error of 174,293 rials. Replacing white noise with mixed noise considerably improved the prediction, producing an estimated price of 794,487 rials and reducing the absolute error to 31,687 rials. The model driven by Lévy noise yielded the most accurate forecast, estimating the exchange rate at 751,867 rials, with an absolute error of only 10,933 rials. Thus, the error of the mixed-noise model was markedly lower than that of the conventional Gaussian model, and the Lévy-noise model further reduced the prediction error relative to both alternatives. The confidence interval calculated from the stochastic Black–Scholes framework also contained the observed exchange-rate value, indicating that the proposed stochastic formulation provided a reasonable representation of uncertainty around the predicted price. Collectively, the findings show that the choice of stochastic driver is a decisive component of exchange-rate modeling and that Lévy noise provides the highest predictive efficiency for a currency series characterized by abrupt changes and discontinuities.

The poor performance of the Gaussian white-noise model can primarily be explained by the restrictive distributional assumptions underlying geometric Brownian motion. In the conventional Black–Scholes model, the stochastic component is represented by a Wiener process with continuous sample paths, independent increments, and normally distributed changes. This formulation is theoretically appropriate when market movements arise from the accumulation of numerous small and approximately symmetric shocks. However, the Iranian exchange-rate series examined in the study included pronounced increases, irregular fluctuations, and sudden shifts that were inconsistent with the smooth continuity implied by Brownian motion. Standard stochastic calculus shows that although a Wiener-driven model can effectively represent continuous diffusion, it cannot directly generate discontinuous jumps unless additional components are introduced [5, 6]. Therefore, when substantial exchange-rate changes occur within short intervals, a Gaussian diffusion model may systematically misrepresent both the frequency and magnitude of extreme observations. The overestimation produced by the white-noise model in this study is consistent with this mismatch between the probabilistic assumptions of the model and the empirical behavior of the exchange-rate data.

The improved performance of the mixed-noise specification suggests that the exchange-rate dynamics were influenced by more than one source of stochastic variation. Mixed noise, operationalized as a weighted linear combination of independent Wiener processes, can represent the simultaneous influence of several random factors. In the context of currency markets, such factors may include macroeconomic expectations, changes in liquidity, monetary policy signals, informal market activity, political announcements, and speculative trading. Although each component remains Gaussian, their combination can provide greater flexibility in representing heterogeneous sources of continuous uncertainty. The reduction of the absolute prediction error from 174,293 rials under white noise to 31,687 rials under mixed noise indicates that treating uncertainty as multidimensional rather than homogeneous substantially improved the approximation of the observed price trajectory.

This interpretation is aligned with previous applications of mixed-noise stochastic models. Farnoosh and colleagues examined an RL electrical circuit under alternative stochastic perturbations and found that a model incorporating different noise terms could more accurately represent the system response than a formulation based on a single conventional stochastic source [8]. Although an electrical circuit differs substantively from a financial market, the methodological principle is transferable: when a dynamic system is exposed to several uncertain influences, a single noise process may be insufficient to describe its variation. Similarly, the use of mixture noise in stochastic exponential growth models has demonstrated that alternative noise structures affect both point estimates and confidence intervals, reinforcing the importance of matching the stochastic specification to the underlying sources of uncertainty [9]. The theoretical treatment of stochastic differential equations also emphasizes that the diffusion term is not merely a technical residual; rather, it expresses substantive assumptions about the probabilistic mechanism driving the system [10]. Accordingly, the superior performance of mixed noise over white noise in the current study reflects the advantage of representing exchange-rate uncertainty through several interacting stochastic components.

Nevertheless, the mixed-noise model remained less accurate than the model incorporating Lévy noise. This result is theoretically expected because a linear combination of independent Gaussian processes remains Gaussian and ordinarily retains continuous sample paths. Therefore, although mixed noise can accommodate several sources of volatility, it does not fully resolve the inability of Brownian motion to reproduce discontinuities and extreme movements. The Iranian exchange-rate market is frequently exposed to shocks arising from geopolitical developments, economic sanctions, expectations regarding international negotiations, inflationary pressures, regulatory interventions, and sudden changes in demand for foreign currency. Such shocks may cause the exchange rate to move sharply rather than gradually. A continuous Gaussian model can respond only by increasing diffusion variance, but this may lead to exaggerated fluctuations throughout the entire simulation rather than generating isolated jumps at appropriate points. This mechanism may explain why the mixed-noise model substantially improved the forecast but still produced an error almost three times larger than that of the Lévy-noise model.

The superior predictive performance of the Lévy specification constitutes the principal finding of the study. Lévy processes generalize Brownian motion by retaining stationary and independent increments while permitting discontinuous jumps and a broader range of increment distributions. Through their drift, Brownian, and pure-jump components, Lévy processes can represent routine fluctuations and exceptional shocks within a unified framework. This property is especially relevant for financial data, which frequently exhibit skewness, excess kurtosis, heavy tails, and abrupt price changes. In the present study, the relatively small error of 10,933 rials under Lévy noise indicates that the observed exchange-rate trajectory was more consistent with a jump-diffusion structure than with a purely continuous Gaussian process. In substantive terms, the result suggests that extreme exchange-rate changes were not simply larger realizations of ordinary volatility; rather, they represented distinct jump-like events that required a non-Gaussian stochastic mechanism.

The finding is consistent with Mohammadi and Nabati's application of a combined Ornstein–Uhlenbeck process with Lévy noise to financial-market modeling. Their results showed that incorporating Lévy disturbances produced simulated fluctuations that remained close to observed market behavior and generated stable estimates around actual values [13]. Although the present study used the Black–Scholes framework rather than a mean-reverting Ornstein–Uhlenbeck process, both studies support the proposition that Lévy noise can improve financial modeling when observed data contain sudden and non-Gaussian changes. The convergence of these findings across different stochastic structures indicates that the advantage of Lévy noise is not restricted to one particular drift formulation. Instead, it arises from the capacity of the driving process to accommodate discontinuities and heavy-tailed innovations.

The present findings are also supported by methodological developments in the simulation of generalized Lévy processes. Godsill and Kindap developed point-process procedures for simulating generalized inverse Gaussian processes and estimating the Jaeger integral, thereby facilitating the computational treatment of flexible non-Gaussian processes [11]. Kindap and Godsill subsequently extended this approach to generalized hyperbolic Lévy processes, which are capable of representing asymmetry and heavy-tailed behavior commonly observed in financial data [12]. These contributions demonstrate that jump-based modeling is not merely a theoretical alternative to Gaussian diffusion but a computationally implementable framework for complex stochastic systems. The results of the current study provide an applied illustration of this principle: even a comparatively direct Euler–Maruyama simulation produced a substantial improvement when the Gaussian driver was replaced by Lévy noise.

The findings further correspond with earlier applications of stochastic differential equations to commodity-price forecasting. Farnoosh and colleagues used stochastic differential equations to simulate and forecast OPEC oil prices, emphasizing the ability of stochastic models to represent price dynamics affected by economic and political uncertainty [14]. Oil prices and exchange rates are both highly sensitive to geopolitical developments, policy decisions, market expectations, and external shocks. Therefore, the success of stochastic differential-equation approaches in commodity markets supports their application to currency forecasting. The present study extends this line of research by demonstrating that the type of noise used in the stochastic model is as important as the general choice to employ an SDE. Merely introducing stochasticity does not guarantee accurate forecasting; the distributional structure of that stochasticity must correspond to the empirical behavior of the market.

From a numerical perspective, the results also confirm the usefulness of the Euler–Maruyama method as an accessible procedure for comparing stochastic model specifications. The method approximates the evolution of the price process by iteratively combining the estimated drift contribution with noise-dependent stochastic increments. Its simplicity permits the simulation of multiple trajectories and facilitates the comparison of prediction errors across alternative stochastic drivers. The numerical theory developed by Kloeden and Platen establishes Euler–Maruyama as a foundational method for the approximation of stochastic differential equations under standard regularity conditions [15]. Carleti and colleagues similarly demonstrated the utility of numerical simulation for stochastic ordinary differential equations in biomathematical settings, showing that stochastic models can be effectively investigated even when analytical solutions are unavailable or impractical [7]. In the current study, the method was sufficiently sensitive to reveal clear differences among white, mixed, and Lévy noise specifications.

At the same time, the results should not be interpreted as implying that machine-learning approaches are irrelevant to exchange-rate forecasting. Recent evidence shows that ensemble machine-learning algorithms can provide strong predictive performance by combining multiple learners and capturing nonlinear relations in highly volatile markets [2]. Comparative studies have likewise found substantial differences among machine-learning models in cryptocurrency price forecasting, with predictive accuracy depending on the algorithm, features, time horizon, and evaluation criteria [1]. Deep-learning approaches have also demonstrated potential for high-frequency, short-term price prediction because they can process complex temporal patterns and large quantities of market information [3]. In addition, machine-learning price forecasts can be incorporated into risk-adjusted portfolio optimization, connecting predictive accuracy with practical financial decision-making [4]. However, these methods primarily learn empirical associations from data, whereas stochastic differential equations explicitly describe the probabilistic mechanisms of drift, volatility, diffusion, and jumps. The current findings therefore support Lévy-driven stochastic modeling as an interpretable alternative or complement to machine learning, rather than as a universally exclusive forecasting method.

Another important implication concerns the role of confidence intervals. The inclusion of the actual exchange-rate value within the estimated interval indicates that the stochastic model provided a plausible range for future price uncertainty. In currency forecasting, a point estimate alone may be insufficient because decision-makers must also understand the range of possible outcomes. Confidence or prediction intervals can support the evaluation of downside risk, budgeting, hedging, and exposure management. The fact that the observed value fell within the calculated interval suggests that the estimated drift and diffusion parameters were reasonably calibrated to the empirical series. However, the substantially different point-prediction errors across noise types also show that acceptable interval coverage does not necessarily imply equal model efficiency. A wide interval can include the realized value even when the central forecast is inaccurate. Therefore, interval coverage should be evaluated alongside absolute error, interval width, and stability across multiple forecast horizons.

Overall, the study demonstrates that exchange-rate forecasting should not rely automatically on the Gaussian assumptions embedded in the standard Black–Scholes model. The progression of prediction errors—from 174,293 rials under white noise, to 31,687 rials under mixed noise, and finally to 10,933 rials under Lévy noise—provides clear empirical evidence that increasing the stochastic realism of the model improved predictive performance. Mixed noise captured multiple continuous sources of uncertainty, while Lévy noise additionally accounted for discontinuous market shocks. The findings consequently support the use of jump-based stochastic models when exchange-rate data display sudden movements, heavy tails, or departures from normality. They also underline a broader modeling principle: the efficiency of a stochastic differential equation depends not only on its drift and diffusion coefficients but also on whether the selected driving process adequately represents the structure of uncertainty observed in the data.

This study has several limitations that should be considered when interpreting its findings. First, the empirical analysis was based on a relatively short period of daily exchange-rate observations, which may not represent all volatility regimes, structural changes, and seasonal patterns in the Iranian currency market. Second, predictive efficiency was assessed using one principal out-of-sample target and absolute prediction error; therefore, the reported superiority of Lévy noise should be examined across additional prediction dates and error criteria. Third, the study focused on a single exchange rate and did not examine whether similar results would be obtained for other currencies, commodities, stocks, or cryptocurrency series. Fourth, the parameters were treated as constant over the observation and prediction periods, although drift, diffusion, and jump intensity may change over time. Fifth, the analysis did not explicitly compare alternative Lévy distributions or estimate separate parameters for jump frequency and jump magnitude. Finally, political, macroeconomic, and market variables were not directly entered into the model, and their effects were represented indirectly through the stochastic noise component.

Future studies should employ longer datasets covering periods of relative stability, economic crisis, policy intervention, and geopolitical shock. Rolling-window and expanding-window evaluations would allow researchers to determine whether the relative performance of white, mixed, and Lévy noise remains stable over time. Multiple out-of-sample forecasts should be generated at short-, medium-, and long-term horizons, and model performance should be evaluated using mean absolute error, root mean squared error, mean absolute percentage error, interval coverage, and directional accuracy. Future research should also compare different members of the Lévy family, including variance-gamma, normal inverse Gaussian, tempered stable, and generalized hyperbolic processes. Time-varying drift, stochastic volatility, regime-switching structures, and jump-diffusion models could further improve realism. Hybrid approaches combining Lévy-driven stochastic differential equations with machine learning may also be developed so that structural probabilistic interpretation and nonlinear data-driven prediction can be integrated within a single framework.

For practical application, analysts should examine exchange-rate returns for jumps, heavy tails, skewness, and departures from normality before selecting a forecasting model. Conventional Gaussian Black–Scholes specifications should be used cautiously when the observed series contains abrupt movements. Financial institutions, importers, exporters, investors, and policymakers may obtain more realistic forecasts by incorporating Lévy noise into stochastic simulations, particularly during periods of political or economic instability. Forecasts should be accompanied by prediction intervals and scenario analyses rather than presented as precise point estimates. Model parameters should be recalibrated frequently as new observations become available, and forecasts should be validated against realized prices over several horizons. In operational settings, Lévy-based models may be implemented alongside econometric and machine-learning methods, with decisions based on comparative out-of-sample performance rather than reliance on a single forecasting framework.

Authors' Contributions

Authors equally contributed to this article.

Ethical Considerations

All procedures performed in this study were under the ethical standards.

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Conflict of Interest

The authors report no conflict of interest.

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